RELATIVISTIC QUANTUM MECHANICS

Tuesday 07-04-2015, 14.00-17.00

On the first sheet write your name, address and student number. Write your name on **all** other sheets. The total number of points is 90. Use conventions with $\hbar = c = 1$.

PROBLEM 1: LORENTZ GROUP (5+5+5=15 points)

1.1 What is the dimension of the scalar, vector and spinor representations of the Lorentz group?

1.2 Which truncations are possible of the spinor representation? Explain these truncations in words: what constraints should one impose? What are the dimensions of the resulting representations?

1.3 How many inequivalent representations does the Lorentz group have? How can these be labelled?

PROBLEM 2: NOETHER'S THEOREM (5+5+15=25 points)

2.1 Describe Noether's theorem in general.

2.2 What does Noether's theorem imply for space translations, time translations and rotations, respectively?

2.3 In the case of time translations, derive the resulting expression for the case of a massive spinor field.

PROBLEM 3: CANONICAL QUANTIZATION (5+5+10+5=25 points)

3.1 Explain the difference between the Schrodinger and Heisenberg pictures of quantization.

3.2 What is the relation between the ϕ operator and the ladder operators in a massive scalar field theory? Give expressions for both the Schrodinger and Heisenberg picture.

3.3 Calculate the action of the differential operator $\Box = \partial_\mu \partial^\mu$ on the expression for the ϕ operator in the Heisenberg picture. Interpret your result.

3.4 What is the analogous expression for the expansion of a relativistic massive spinor field in terms of ladder operators? Give the result both in the Schrodinger and Heisenberg picture.

PROBLEM 4: MOMENTUM OPERATOR (15+5+5=25 points)

The momentum operator for a relativistic massive scalar field ϕ is given by

$$\vec{P} = \int d^3x \, \dot{\phi} \vec{\nabla} \phi \, .$$

4.1 Derive the expression for the momentum operator in terms of ladder operators. Interpret your result.

4.2 Calculate the action of $\vec{P}$ on $|\vec{p}\,>$, where the latter is defined as $a^\dagger_{\vec{p}}|0>$. Interpret your result.

4.3 Calculate the action of $\vec{P}$ on $|\vec{p}, \vec{q}\,>$, where the latter is defined as $a^\dagger_{\vec{p}} a^\dagger_{\vec{q}} |0>$. Interpret your result.